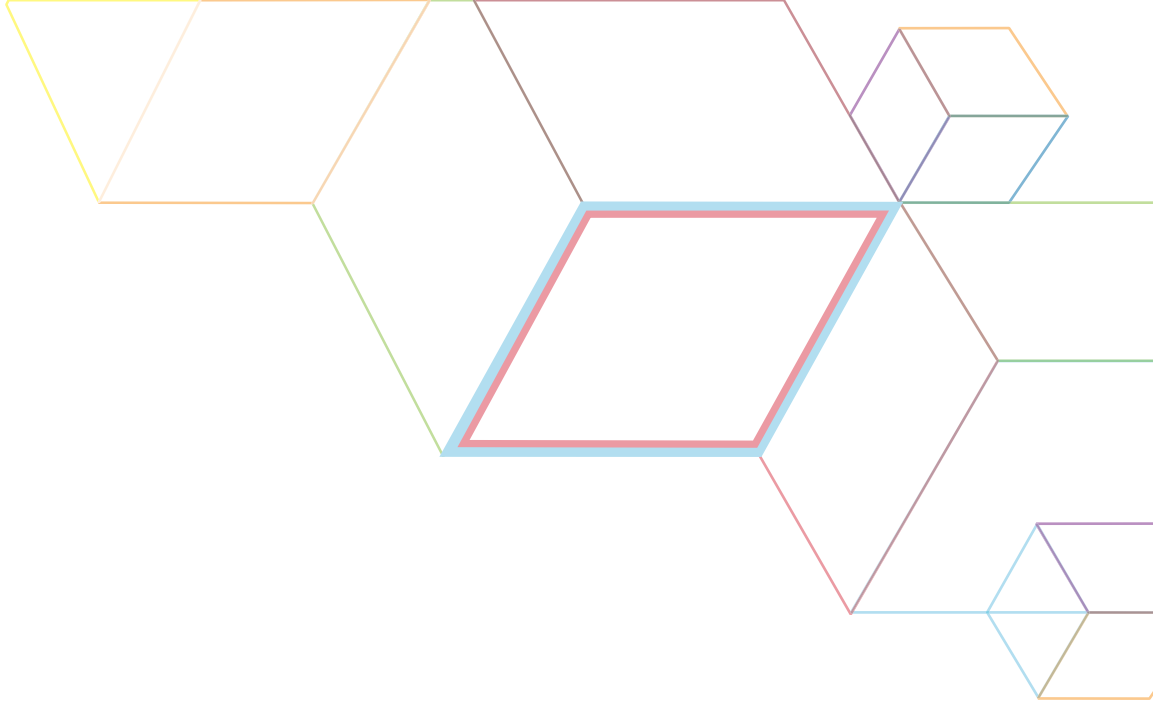
THINKING MATHEMATICALLY

6TH EDITION

Blitzer



Thinking Mathematically

Sixth Edition

Robert Blitzer
Miami Dade College

PEARSON

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About the Author

Bob Blitzer is a native of Manhattan and received a Bachelor of Arts degree with dual majors in mathematics and psychology (minor: English literature) from the City College of New York. His unusual combination of academic interests led him toward a Master of Arts in mathematics from the University of Miami and a doctorate in behavioral sciences from Nova University. Bob's love for teaching mathematics was nourished for nearly 30 years at Miami Dade College, where he received numerous teaching awards, including Innovator of the Year from the League for Innovations in the Community College and an endowed chair based on excellence in the classroom. In addition to *Thinking Mathematically*, Bob has written textbooks covering introductory algebra, intermediate algebra, college algebra, algebra and trigonometry, precalculus, trigonometry, and liberal arts mathematics for high school students, all published by Pearson. When not secluded in his Northern California writer's cabin, Bob can be found hiking the beaches and trails of Point Reyes National Seashore, and tending to the chores required by his beloved entourage of horses, chickens, and irritable roosters.

Bob and his buddy Casper Cockatoo

Preface

Thinking Mathematically, Sixth Edition provides a general survey of mathematical topics that are useful in our contemporary world. My primary purpose in writing the book was to show students how mathematics can be applied to their lives in interesting, enjoyable, and meaningful ways. The book's variety of topics and flexibility of sequence make it appropriate for a one- or two-term course in liberal arts mathematics, quantitative reasoning, finite mathematics, as well as for courses specifically designed to meet state-mandated requirements in mathematics.

I wrote the book to help diverse students, with different backgrounds and career plans, to succeed. **Thinking Mathematically**, **Sixth Edition**, has four major goals:

1. To help students acquire knowledge of fundamental mathematics.
2. To show students how mathematics can solve authentic problems that apply to their lives.
3. To enable students to understand and reason with quantitative issues and mathematical ideas they are likely to encounter in college, career, and life.
4. To enable students to develop problem-solving skills, while fostering critical thinking, within an interesting setting.

One major obstacle in the way of achieving these goals is the fact that very few students actually read their textbook. This has been a regular source of frustration for me and my colleagues in the classroom. Anecdotal evidence gathered over years highlights two basic reasons why students do not take advantage of their textbook:

"I'll never use this information."
"I can't follow the explanations."

I've written every page of the Sixth Edition with the intent of eliminating these two objections. The ideas and tools I've used to do so are described for the student in "A Brief Guide to Getting the Most from This Book," which appears inside the front cover.

What's New in the Sixth Edition?

- **New Applications and Real-World Data.** I'm on a constant search for real-world data that can be used to illustrate unique mathematical applications. I researched hundreds of books, magazines, newspapers, almanacs, and online sites to prepare the Sixth Edition. This edition contains 366 worked-out examples and application exercises based on new data sets.
- **Concept and Vocabulary Checks.** The Sixth Edition contains 653 new short-answer exercises, mainly fill-in-the-blank and true/false items, that assess students' understanding of the definitions and concepts presented in each section. The Concept and Vocabulary Checks appear as separate features preceding the Exercise Sets.
- **Great Question!** This feature takes the content of each Study Tip in the Fifth Edition and presents it in the context of a student question. Answers to questions offer suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions. 'Great Question!' should draw students' attention and curiosity more than the 'Study Tips.' As a secondary benefit, this new feature should help students not to feel anxious or threatened when asking questions in class. The feature is extended to the learning objectives at the beginning of each section, which are now framed in the context of a student question: What am I supposed to learn?
- **New Blitzer Bonuses.** The Sixth Edition contains a variety of new but optional enrichment essays. There are more new Blitzer Bonuses in this edition than in any previous revision of *Thinking Mathematically*. These include "Are You Smart Enough to Work at Google?" (Section 1.1), "Science and Math Tattoos" (Section 2.1), "NUMB3RS: Solving Crime with Mathematics" (Section 5.3), "Using Algebra to Measure Blood-Alcohol Concentration" (Section 6.1), "Testing Your Financial Literacy" (Section 8.1), "The Bottom Line on Investments" (Section 8.5), "Financing Your Car" (Section 8.6), "Reducing Rental Costs" (Section 8.7), "College Students and Credit Cards" (Section 8.8), "Big Fears and Their Odds" (Section 11.6), "Using Means to Compare How the U.S. Measures Up" (Section 12.2), "The 2012 Presidential Election" (Section 13.4), and "A Family Tree: The Sopranos" (Section 14.4).
- **Brief Reviews.** The book's Brief Review boxes summarize mathematical skills that students should have learned previously, but which many students still need to review. This feature appears whenever a particular skill is first needed and eliminates the need to reteach that skill.
- **Sample Homework Assignments.** Within each Exercise Set, I have chosen odd-numbered problems from the Practice Exercises and the Application Exercises that can serve as sample homework assignments. These are indicated by a red underline in the Annotated Instructor's Edition. Based on the goals and objectives of your course, you may wish to enrich each sample homework assignment with additional exercises from the other categories in the Exercise Set.

- **Learning Guide.** This study aid is organized by objective and provides support for note-taking, practice, and video review. The Learning Guide is available as PDFs and customizable Word files in MyMathLab. They can also be packaged with the textbook and MyMathLab access code.
- **Thinking Mathematically with Integrated Review.** For courses where students do require more extensive prerequisite review, we have created a version of the *Thinking Mathematically* MyMathLab course called *Thinking Mathematically with Integrated Review* that includes just-in-time review of select topics where appropriate. Students are asked to check their skills with an assignment at the start of each chapter to assess their understanding of requisite, developmental material. For those students who do require further review, resources include the eText, short objective-based videos, Integrated Review Worksheets, and Integrated Review Homework to help provide students with a solid foundation on the review topics needed for their *Thinking Mathematically* course.

What Content and Organizational Changes Have Been Made to the Sixth Edition?

- **Section 3.3 (Truth Tables for Negation, Conjunction, and Disjunction)** opens with a new application on the distribution of looks for U.S. men and women. The application reappears in an example on determining the truth value of a compound statement.
- **Section 4.2 (Number Bases in Positional Systems)** contains a new discussion (within the context of the Great Question! feature) on the use of octal and hexadecimal systems by computer programmers.
- **Section 5.5 (Real Numbers and Their Properties; Clock Addition)** integrates material from Chapter 13 (Mathematical Systems) of the Fifth Edition by applying properties of real numbers to clock arithmetic and discussing related symmetries.
- **Section 6.3 (Applications of Linear Equations)** has a new example on the starting salaries for college graduates with undergraduate degrees. The example from the Fifth Edition on comparing long-distance telephone plans has been replaced by an example on choosing between texting plans.
- **Chapter 8 (Personal Finance)** has been renamed and expanded to include relevant information, both mathematical and non-mathematical, to help students manage their finances. The Sixth Edition contains separate sections on income tax, cars, home ownership, and credit cards.
- **Section 8.2 (Income Tax)** expands the discussion of income tax from the Fifth Edition, giving a broader understanding of the terms and complexities involved in calculating taxes. Included in this new section are discussions of Social Security and Medicare (FICA), as well as an example related to taxes for working students.
- **Section 8.6 (Cars)** is a new section that uses the mathematics of financing a car to develop the loan payment formula for fixed installment loans. The section includes new objectives on the pros and cons of leasing versus buying a car, understanding the different kinds of car insurance, comparing monthly payments on new and used cars, and solving problems related to owning and operating a car.
- **Section 8.7 (The Cost of Home Ownership)** is a new section that applies the loan payment formula that was developed for cars in Section 8.6 to mortgages. The section includes new objectives on solving problems involving what one can afford to spend for a mortgage, and understanding the pros and cons of renting versus buying.
- **Section 8.8 (Credit Cards)** is a new section devoted entirely to credit cards. Objectives unique to the Sixth Edition include understanding the pros and cons of using credit cards, understanding the difference between credit cards and debit cards, knowing what is contained in a credit report, and understanding credit scores as measures of creditworthiness.
- **Section 9.2 (Measuring Area and Volume)** and **Section 9.3 (Measuring Weight and Temperature)** incorporate new discussions on measuring dosages of medication, including dosages based on weight in the metric system.
- **Section 12.4 (The Normal Distribution)** uses the activities U.S. adults say they dread to illustrate a poll's margin of error.

What Familiar Features Have Been Retained in the Sixth Edition?

- **Chapter-Opening and Section-Opening Scenarios.** Every chapter and every section open with a scenario presenting a unique application of mathematics in students' lives outside the classroom. These scenarios are revisited in the course of the chapter or section in an example, discussion, or exercise. The often humorous tone of these openers is intended to help fearful and reluctant students overcome their negative perceptions about math. A new feature in the Sixth Edition, "Here's Where You'll Find These Applications," is included with each chapter opener.
- **Detailed Worked-Out Examples.** Each example is titled, making the purpose of the example clear. Examples are clearly written and provide students with detailed step-by-step solutions. No steps are omitted and each step is thoroughly explained to the right of the mathematics.
- **Explanatory Voice Balloons.** Voice Balloons are used in a variety of ways to demystify mathematics. They translate mathematical language into everyday English, help clarify problem-solving procedures, present alternative ways of understanding concepts, and connect problem solving to concepts students have already learned.

- **Check Point Examples.** Each example is followed by a similar matched problem, called a Check Point, offering students the opportunity to test for conceptual understanding by working a similar exercise. The answers to the Check Points are provided in the answer section in the back of the book. Worked-out video solutions are in the MyMathLab courses or on YouTube.
- **Extensive and Varied Exercise Sets.** An abundant collection of exercises is included in an Exercise Set at the end of each section. Exercises are organized within seven category types: Practice Exercises, Practice Plus Exercises, Application Exercises, Writing in Mathematics, Critical Thinking Exercises, Technology Exercises, and Group Exercises.
- **Practice Plus Problems.** This category of exercises contains practice problems that often require students to combine several skills or concepts, providing instructors the option of creating assignments that take Practice Exercises to a more challenging level.
- **Section Objectives (What Am I Supposed to Learn?).** Learning objectives are clearly stated at the beginning of each section. These objectives help students recognize and focus on the section's most important ideas. The objectives are restated in the margin at their point of use.
- **Chapter Summaries.** Each chapter contains a review chart that summarizes the definitions and concepts in every section of the chapter. Examples that illustrate these key concepts are also referenced in the chart.
- **End-of-Chapter Materials.** A comprehensive collection of review exercises for each of the chapter's sections follows the Summary. This is followed by a Chapter Test that enables students to test their understanding of the material covered in the chapter. Worked-out video solutions are available for every Chapter Test Prep problem in the MyMathLab course or on YouTube.

I hope that my love for learning, as well as my respect for the diversity of students I have taught and learned from over the years, is apparent throughout this new edition. By connecting mathematics to the whole spectrum of learning, it is my intent to show students that their world is profoundly mathematical, and indeed, π is in the sky.

Robert Blitzer

Dynamic Resources

MyMathLab (access code required)

MyMathLab from Pearson is the world's leading online resource in mathematics, integrating interactive homework, assessment, and media in a flexible, easy to use format. It provides **engaging experiences** that personalize, stimulate, and measure learning for each student. And, it comes from an **experienced partner** with educational expertise and an eye on the future.

To learn more about how MyMathLab combines proven learning applications with powerful assessment, visit **www.mymathlab.com** or contact your Pearson representative.

Blitzer's MyMathLab course provides access to innovative and engaging study solutions to increase student success.

Ready to Go Course

This new MyMathLab course option provides students with all the same great MyMathLab features, but makes it easier for instructors to get started, with premade and pre-assigned assignments.

Integrated Review Course Solution

For courses where students require more extensive remediation, we have created the Integrated Review solution that includes just-in-time review of select topics where appropriate. For students who require this review, resources include the eText, short objective-based videos, Integrated Review Worksheets, and Integrated Review Homework to help provide students with a solid foundation for success in their *Thinking Mathematically* course.

Getting Ready

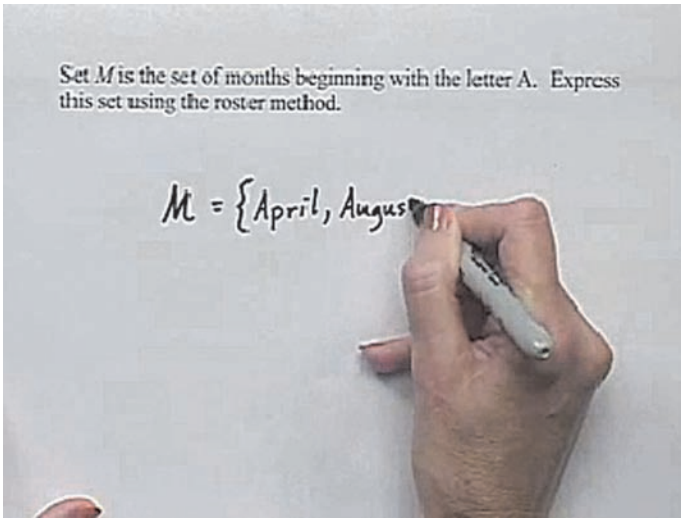
Getting Ready exercises are now available for online review in the Standard and Ready to Go MyMathLab courses. These skill review quizzes test on prerequisite knowledge, allowing students to refresh forgotten concepts.

MathTalk Videos

These fun, application-based videos connect the math in Blitzer to the real world. These instructional videos have a sense of humor while demonstrating how everyday life is full of math applications.

Concept and Vocabulary Check

New Concept and Vocabulary Check exercises provide a quick check of understanding of concepts. Assignable in MyMathLab, these also test for reading comprehension before moving onto the homework exercise sets.



Check Point Videos

These videos show instructors working out every Check Point problem in the text to ensure understanding. New to the Sixth Edition MyMathLab course are assignable Check Point exercises that correspond with each video, ensuring that students watched the video and understood the concepts presented.

Chapter Test Prep Videos

Students can watch instructors work through step-by-step solutions to all the Chapter Test Prep exercises from the textbook. These are available in MyMathLab and on YouTube.

Student Success Module

A new Student Success Module supports students toward continued success in college. This module provides tutorials and guidance on topics such as transition to college, online learning, time management, and more. Additionally, there is content to help students develop professional skills such as resume development and interview preparation.

Instructor Resources

Annotated Instructor's Edition

ISBN-13: 978-0321-91487-3
ISBN-10: 0-321-91487-2

Additional resources can be downloaded from www.pearsonhighered.com:

TestGen

Powerpoint Lecture Slides

Instructor's Solutions Manual

Instructor's Testing Manual

Student Resources

Learning Guide – NEW!

This study aid is organized by objective and provides support for note-taking, practice, and video review. The Learning Guide is available as PDFs and customizable Word files in MyMathLab. It can also be packaged with the textbook and MyMathLab access code.

Student's Solutions Manual

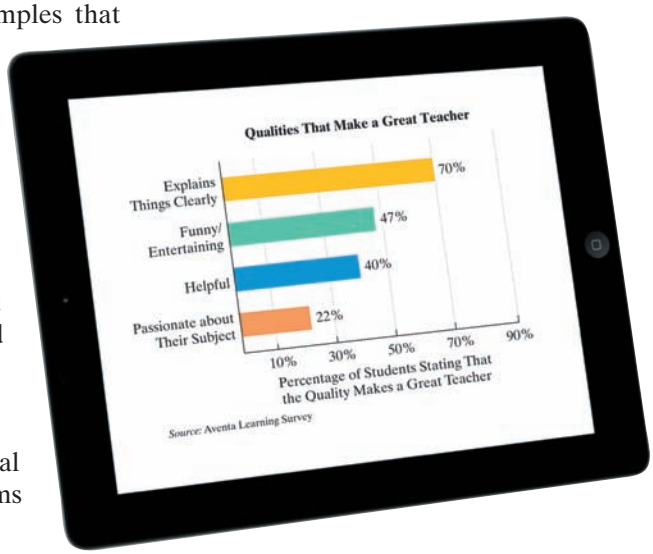
This manual contains fully worked solutions to odd-numbered exercises and all Check Points.

To the Student

The bar graph shows some of the qualities that students say make a great teacher. It was my goal to incorporate each of these qualities throughout the pages of this book to help you gain control over the part of your life that involves numbers and mathematical ideas.

Explains Things Clearly

I understand that your primary purpose in reading *Thinking Mathematically* is to acquire a solid understanding of the required topics in your liberal arts math course. In order to achieve this goal, I've carefully explained each topic. Important definitions and procedures are set off in boxes, and worked-out examples that present solutions in a step-by-step manner appear in every section. Each example is followed by a similar matched problem, called a Check Point, for you to try so that you can actively participate in the learning process as you read the book. (Answers to all Check Points appear in the back of the book and video solutions are in MyMathLab.)



Funny & Entertaining

Who says that a math textbook can't be entertaining? From our quirky cover to the photos in the chapter and section openers, prepare to expect the unexpected. I hope some of the book's enrichment essays, called Blitzer Bonuses, will put a smile on your face from time to time.

Helpful

I designed the book's features to help you acquire knowledge of fundamental mathematics, as well as to show you how math can solve authentic problems that apply to your life. These helpful features include

- **Explanatory Voice Balloons:** Voice balloons are used in a variety of ways to make math less intimidating. They translate mathematical language into everyday English, help clarify problem-solving procedures, present alternative ways of understanding concepts, and connect new concepts to concepts you have already learned.
- **Great Question!:** The book's Great Question! boxes are based on questions students ask in class. The answers to these questions give suggestions for problem solving, point out common errors to avoid, and provide informal hints and suggestions.
- **Chapter Summaries:** Each chapter contains a review chart that summarizes the definitions and concepts in every section of the chapter. Examples from the chapter that illustrate these key concepts are also referenced in the chart. Review these summaries and you'll know the most important material in the chapter!

Passionate about the Subject

I passionately believe that no other discipline comes close to math in offering a more extensive set of tools for application and development of your mind. I wrote the book in Point Reyes National Seashore, 40 miles north of San Francisco. The park consists of 75,000 acres with miles of pristine surf-washed beaches, forested ridges, and bays bordered by white cliffs. It was my hope to convey the beauty and excitement of mathematics using nature's unspoiled beauty as a source of inspiration and creativity. Enjoy the pages that follow as you empower yourself with the mathematics needed to succeed in college, your career, and in your life.

Regards,

Bob

Robert Blitzer

Acknowledgments

An enormous benefit of authoring a successful textbook is the broad-based feedback I receive from students, dedicated users, and reviewers. Every change to this edition is the result of their thoughtful comments and suggestions. I would like to express my appreciation to all the reviewers, whose collective insights form the backbone of this revision. In particular, I would like to thank the following people for reviewing *Thinking Mathematically* for this Sixth Edition.

Deana Alexander, *Indiana University—Purdue University*
Nina Bohrod, *Anoka-Ramsey Community College*
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Cindy Vanderlaan, *Indiana Purdue University—Fort Wayne*
Alexandra Verkhovtseva, *Anoka-Ramsey Community College*

Each reviewer from every edition has contributed to the success of this book and I would like to also continue to offer my thanks to them.

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superbly illustrating the book; and Rebecca Dunn, project manager, and Kathleen Manley, production editor, whose collective talents kept every aspect of this complex project moving through its many stages, and Kerri Consalvo who helped keep everything organized.

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Robert Blitzer

Index of Applications

D

E

F

G

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1

Problem Solving and Critical Thinking

HOW WOULD YOUR LIFESTYLE CHANGE IF A GALLON OF GAS COST $9.15? OR IF THE PRICE OF A staple such as milk was $15? That's how much those products would cost if their prices had increased at the same rate college tuition has increased since 1980.

TUITION AND FEES AT FOUR-YEAR COLLEGES

	School Year Ending 2000	School Year Ending 2010
Public	$3362	$7020
Private	$15,518	$26,273

Source: The College Board

If these trends continue, what can we expect for the rest of this decade and beyond? We can answer this question by using estimation techniques that allow us to represent the data mathematically. With such representations, called *mathematical models*, we can gain insights and predict what might occur in the future on a variety of issues, ranging from college costs to global warming.

Here's where you'll find these applications:

Mathematical models involving college costs are developed in Example 8 and Check Point 8 of Section 1.2. In Exercises 51 and 52 in Exercise Set 1.2, you will approach our climate crisis mathematically by developing models for data related to global warming.

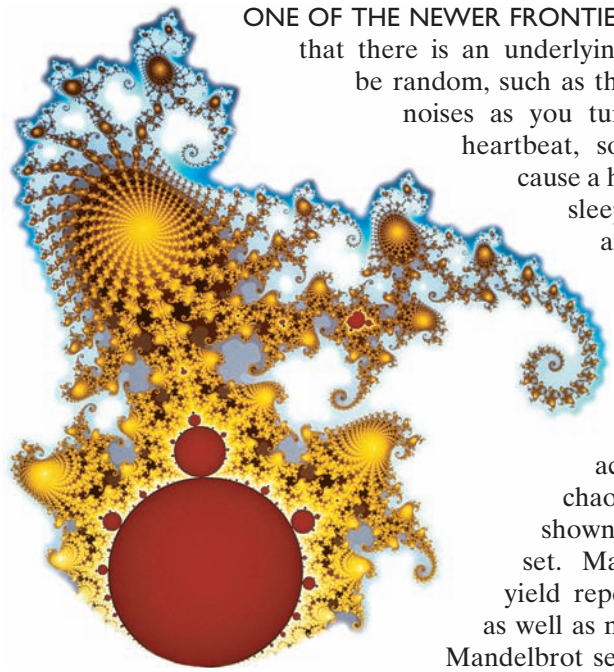
1.1

WHAT AM I SUPPOSED TO LEARN?

After you have read this section, you should be able to:

1. Understand and use inductive reasoning.
2. Understand and use deductive reasoning.

Inductive and Deductive Reasoning



A magnification of the Mandelbrot set
Richard F. Voss

ONE OF THE NEWER FRONTIERS OF MATHEMATICS SUGGESTS that there is an underlying order in things that appear to be random, such as the hiss and crackle of background noises as you tune a radio. Irregularities in the heartbeat, some of them severe enough to cause a heart attack, or irregularities in our sleeping patterns, such as insomnia, are examples of chaotic behavior. Chaos in the mathematical sense does not mean a complete lack of form or arrangement. In mathematics, chaos is used to describe something that appears to be random but is not actually random. The patterns of chaos appear in images like the one shown on the left, called the Mandelbrot set. Magnified portions of this image yield repetitions of the original structure, as well as new and unexpected patterns. The Mandelbrot set transforms the hidden structure of chaotic events into a source of wonder and inspiration.

Many people associate mathematics with tedious computation, meaningless algebraic procedures, and intimidating sets of equations. The truth is that mathematics is the most powerful means we have of exploring our world and describing how it works. The word *mathematics* comes from the Greek word *mathematikos*, which means "inclined to learn." To be mathematical literally means to be inquisitive, open-minded, and interested in a lifetime of pursuing knowledge!

Mathematics and Your Life

A major goal of this book is to show you how mathematics can be applied to your life in interesting, enjoyable, and meaningful ways. The ability to think mathematically and reason with quantitative issues will help you so that you can:

- order and arrange your world by using sets to sort and classify information (Chapter 2, Set Theory)
- use logic to evaluate the arguments of others and become a more effective advocate for your own beliefs (Chapter 3, Logic)
- understand the relationship between cutting-edge technology and ancient systems of number representation (Chapter 4, Number Representation and Calculation)
- put the numbers you encounter in the news, from contemplating the national debt to grasping just how colossal $1 trillion actually is, into perspective (Chapter 5, Number Theory and the Real Number System)
- use mathematical models to gain insights into a variety of issues, including the positive benefits that humor and laughter can have on your life (Chapter 6, Algebra: Equations and Inequalities)

- use basic ideas about savings, loans, and investments to achieve your financial goals (Chapter 8, Personal Finance)
- use geometry to study the shape of your world, enhancing your appreciation of nature's patterns and beauty (Chapter 10, Geometry)
- develop an understanding of the fundamentals of statistics and how these numbers are used to make decisions (Chapter 12, Statistics)
- understand the mathematical paradoxes of voting in a democracy, increasing your ability to function as a more fully aware citizen (Chapter 13, Voting and Apportionment)
- use graph theory to examine how mathematics is used to solve problems in the business world (Chapter 14, Graph Theory)

"It is better to take what may seem to be too much math rather than too little. Career plans change, and one of the biggest roadblocks in undertaking new educational or training goals is poor preparation in mathematics. Furthermore, not only do people qualify for more jobs with more math, they are also better able to perform their jobs."
—Occupational Outlook Quarterly

Mathematics and Your Career

Generally speaking, the income of an occupation is related to the amount of education required. This, in turn, is usually related to the skill level required in language and mathematics. With our increasing reliance on technology, the more mathematics you know, the more career choices you will have.

Mathematics and Your World

Mathematics is a science that helps us recognize, classify, and explore the hidden patterns of our universe. Focusing on areas as different as planetary motion, animal markings, shapes of viruses, aerodynamics of figure skaters, and the very origin of the universe, mathematics is the most powerful tool available for revealing the underlying structure of our world. Within the last 30 years, mathematicians have even found order in chaotic events such as the uncontrolled storm of noise in the nerve cells of the brain during an epileptic seizure.

Understand and use inductive reasoning.

Inductive Reasoning

Mathematics involves the study of patterns. In everyday life, we frequently rely on patterns and routines to draw conclusions. Here is an example:

> The last six times I went to the beach, the traffic was light on Wednesdays and heavy on Sundays. My conclusion is that weekdays have lighter traffic than weekends.

This type of reasoning process is referred to as *inductive reasoning*, or *induction*.

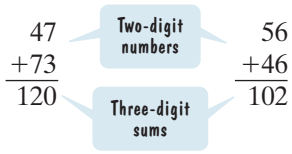
INDUCTIVE REASONING

Inductive reasoning is the process of arriving at a general conclusion based on observations of specific examples.

Although inductive reasoning is a powerful method of drawing conclusions, we can never be absolutely certain that these conclusions are true. For this reason, the conclusions are called **conjectures**, **hypotheses**, or educated guesses. A strong inductive argument does not guarantee the truth of the conclusion, but rather provides strong support for the conclusion. If there is just one case for which the conjecture does not hold, then the conjecture is false. Such a case is called a **counterexample**.

EXAMPLE 1 *Finding a Counterexample*

The ten symbols that we use to write numbers, namely 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9, are called **digits**. In each example shown below, the sum of two two-digit numbers is a three-digit number.



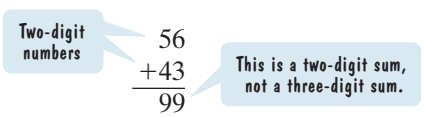
Is the sum of two two-digit numbers always a three-digit number? Find a counterexample to show that the statement

The sum of two two-digit numbers is a three-digit number

is false.

SOLUTION

There are many counterexamples, but we need to find only one. Here is an example that makes the statement false:



This example is a counterexample that shows the statement

The sum of two two-digit numbers is a three-digit number

is false.

GREAT QUESTION!

Why is it so important to work each of the book's Check Points?

You learn best by doing. Do not simply look at the worked examples and conclude that you know how to solve them. To be sure you understand the worked examples, try each Check Point. Check your answer in the answer section before continuing your reading. Expect to read this book with pencil and paper handy to work the Check Points.

CHECK POINT 1 Find a counterexample to show that the statement

The product of two two-digit numbers is a three-digit number

is false.

Here are two examples of inductive reasoning:

- **Strong Inductive Argument** In a random sample of 380,000 freshmen at 722 four-year colleges, 25% said they frequently came to class without completing readings or assignments (*Source*: National Survey of Student Engagement). We can conclude that there is a 95% probability that between 24.84% and 25.15% of all college freshmen frequently come to class unprepared.

In Chapter 12, you will learn how observations from a randomly selected group, one in which each member of the population has an equal chance of being selected, can provide probabilities of what is true about an entire population.

- **Weak Inductive Argument** Neither my dad nor my boyfriend has ever cried in front of me. Therefore, men have difficulty expressing their feelings.

When generalizing from observations about your own circumstances and experiences, avoid jumping to hasty conclusions based on a few observations. Psychologists theorize that we do this - that is, place everyone in a neat category - to feel more secure about ourselves and our relationships to others.

Inductive reasoning is extremely important to mathematicians. Discovery in mathematics often begins with an examination of individual cases to reveal patterns about numbers.

"For thousands of years, people have loved numbers and found patterns and structures among them. The allure of numbers is not limited to or driven by a desire to change the world in a practical way. When we observe how numbers are connected to one another, we are seeing the inner workings of a fundamental concept."

—Edward B. Burger and Michael Starbird, *Coincidences, Chaos, and All That Math Jazz*, W.W. Norton and Company, 2005

EXAMPLE 2 *Using Inductive Reasoning*

Identify a pattern in each list of numbers. Then use this pattern to find the next number.

a. 3, 12, 21, 30, 39, _____

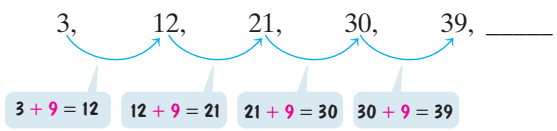
b. 3, 12, 48, 192, 768, _____

c. 3, 4, 6, 9, 13, 18, _____

d. 3, 6, 18, 36, 108, 216, _____

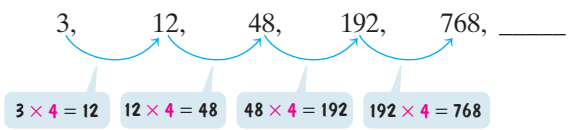
SOLUTION

a. Because 3, 12, 21, 30, 39, _____ is increasing relatively slowly, let's use addition as the basis for our individual observations.



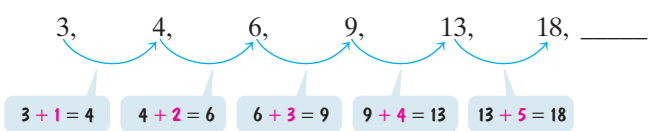
Generalizing from these observations, we conclude that each number after the first is obtained by adding 9 to the previous number. Using this pattern, the next number is 39 + 9, or 48.

b. Because 3, 12, 48, 192, 768, _____ is increasing relatively rapidly, let's use multiplication as the basis for our individual observations.



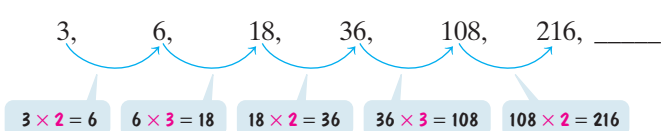
Generalizing from these observations, we conclude that each number after the first is obtained by multiplying the previous number by 4. Using this pattern, the next number is 768 × 4, or 3072.

c. Because 3, 4, 6, 9, 13, 18, _____ is increasing relatively slowly, let's use addition as the basis for our individual observations.



Generalizing from these observations, we conclude that each number after the first is obtained by adding a counting number to the previous number. The additions begin with 1 and continue through each successive counting number. Using this pattern, the next number is 18 + 6, or 24.

d. Because 3, 6, 18, 36, 108, 216, _____ is increasing relatively rapidly, let's use multiplication as the basis for our individual observations.



Generalizing from these observations, we conclude that each number after the first is obtained by multiplying the previous number by 2 or by 3. The multiplications begin with 2 and then alternate, multiplying by 2, then 3, then 2, then 3, and so on. Using this pattern, the next number is 216 × 3, or 648.

CHECK POINT 2 Identify a pattern in each list of numbers. Then use this pattern to find the next number.

a. 3, 9, 15, 21, 27, _____

b. 2, 10, 50, 250, _____

c. 3, 6, 18, 72, 144, 432, 1728, _____

d. 1, 9, 17, 3, 11, 19, 5, 13, 21, _____

In our next example, the patterns are a bit more complex than the additions and multiplications we encountered in Example 2.

EXAMPLE 3 *Using Inductive Reasoning*

Identify a pattern in each list of numbers. Then use this pattern to find the next number.

a. 1, 1, 2, 3, 5, 8, 13, 21, _____ **b.** 23, 54, 95, 146, 117, 98, _____

SOLUTION

a. We begin with 1, 1, 2, 3, 5, 8, 13, 21. Starting with the third number in the list, let's form our observations by comparing each number with the two numbers that immediately precede it.

1, 1, 2, 3, 5, 8, 13, 21, _____

preceded by 1 and 1: $1 + 1 = 2$	preceded by 1 and 2: $1 + 2 = 3$	preceded by 2 and 3: $2 + 3 = 5$	preceded by 3 and 5: $3 + 5 = 8$	preceded by 5 and 8: $5 + 8 = 13$	preceded by 8 and 13: $8 + 13 = 21$

Generalizing from these observations, we conclude that the first two numbers are 1. Each number thereafter is the sum of the two preceding numbers. Using this pattern, the next number is 13 + 21, or 34. (The numbers 1, 1, 2, 3, 5, 8, 13, 21, and 34 are the first nine terms of the *Fibonacci sequence*, discussed in Chapter 5, Section 5.7.)

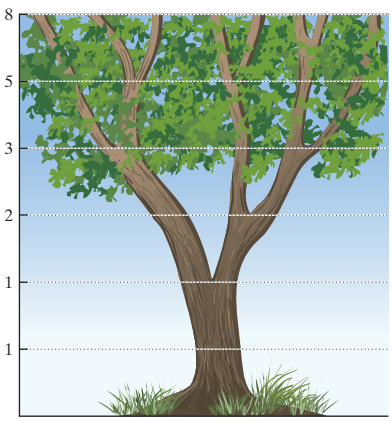
b. Now, we consider 23, 54, 95, 146, 117, 98. Let's use the digits that form each number as the basis for our individual observations. Focus on the sum of the digits, as well as the final digit increased by 1.

$2 + 3 = 5$ | $5 + 4 = 9$ | $9 + 5 = 14$ | $1 + 4 + 6 = 11$ | $1 + 1 + 7 = 9$

23, 54, 95, 146, 117, 98, _____

$3 + 1 = 4$ | $4 + 1 = 5$ | $5 + 1 = 6$ | $6 + 1 = 7$ | $7 + 1 = 8$

Generalizing from these observations, we conclude that for each number after the first, we obtain the first digit or the first two digits by adding the digits of the previous number. We obtain the last digit by adding 1 to the final digit of the preceding number. Applying this pattern to find the number that follows 98, the first two digits are 9 + 8, or 17. The last digit is 8 + 1, or 9. Thus, the next number in the list is 179.



As this tree branches, the number of branches forms the Fibonacci sequence.

GREAT QUESTION!

Can a list of numbers have more than one pattern?

Yes. Consider the illusion in **Figure 1.1**. This ambiguous figure contains two patterns, where it is not clear which pattern should predominate. Do you see a wine goblet or two faces looking at each other? Like this ambiguous figure, some lists of numbers can display more than one pattern, particularly if only a few numbers are given. Inductive reasoning can result in more than one probable next number in a list.

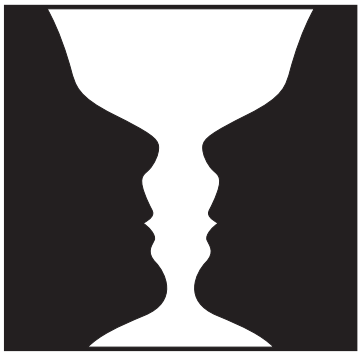


FIGURE 1.1

Example: 1, 2, 4, _____

Pattern: Each number after the first is obtained by multiplying the previous number by 2. The missing number is 4×2, or 8.

Pattern: Each number after the first is obtained by adding successive counting numbers, starting with 1, to the previous number. The second number is $1 + 1$, or 2. The third number is $2 + 2$, or 4. The missing number is $4 + 3$, or 7.

Inductive reasoning can also result in different patterns that produce the same probable next number in a list.

Example: 1, 4, 9, 16, 25, _____

Pattern: Start by adding 3 to the first number. Then add successive odd numbers, 5, 7, 9, and so on. The missing number is $25 + 11$, or 36.

Pattern: Each number is obtained by squaring its position in the list: The first number is $1^2 = 1 \times 1 = 1$, the second number is $2^2 = 2 \times 2 = 4$, the third number is $3^2 = 3 \times 3 = 9$, and so on. The missing sixth number is $6^2 = 6 \times 6$, or 36.

The numbers that we found in Examples 2 and 3 are probable numbers. Perhaps you found patterns other than the ones we pointed out that might have resulted in different answers.

CHECK POINT 3 Identify a pattern in each list of numbers. Then use this pattern to find the next number.

a. 1, 3, 4, 7, 11, 18, 29, 47, ____

b. 2, 3, 5, 9, 17, 33, 65, 129, ____

This electron microscope photograph shows the knotty shape of the Ebola virus.

Mathematics is more than recognizing number patterns. It is about the patterns that arise in the world around us. For example, by describing patterns formed by various kinds of knots, mathematicians are helping scientists investigate the knotty shapes and patterns of viruses. One of the weapons used against viruses is based on recognizing visual patterns in the possible ways that knots can be tied.

Our next example deals with recognizing visual patterns.

EXAMPLE 4 *Finding the Next Figure in a Visual Sequence*

Describe two patterns in this sequence of figures. Use the patterns to draw the next figure in the sequence.

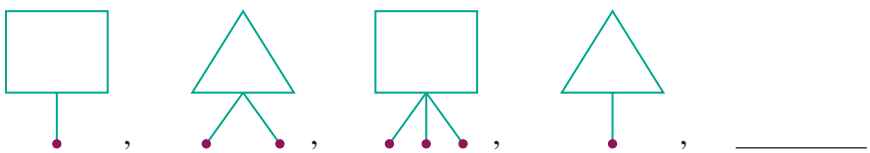
SOLUTION

The more obvious pattern is that the figures alternate between circles and squares. We conclude that the next figure will be a circle. We can identify the second pattern in the four regions containing no dots, one dot, two dots, and three dots. The dots are placed in order (no dots, one dot, two dots, three dots) in a clockwise direction. However, the entire pattern of the dots rotates counterclockwise as we follow the figures from left to right. This means that the next figure should be a circle with a single dot in the right-hand region, two dots in the bottom region, three dots in the left-hand region, and no dots in the top region.

The missing figure in the visual sequence on the previous page, a circle with a single dot in the right-hand region, two dots in the bottom region, three dots in the left-hand region, and no dots in the top region, is drawn in **Figure 1.2.**

FIGURE 1.2

CHECK POINT 4 Describe two patterns in this sequence of figures. Use the patterns to draw the next figure in the sequence.



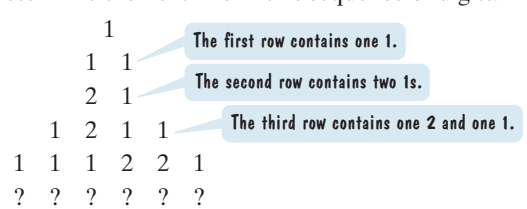
Blitzer Bonus

Are You Smart Enough to Work at Google?

In *Are You Smart Enough to Work at Google?* (Little, Brown, and Company, 2012), author William Poundstone guides readers through the surprising solutions to challenging job-interview questions. The book covers the importance of creative thinking in inductive reasoning, estimation, and problem solving. Best of all, Poundstone explains the answers.

Whether you're preparing for a job interview or simply want to increase your critical thinking skills, we highly recommend tackling the puzzles in *Are You Smart Enough to Work at Google?* Here is a sample of two of the book's problems that involve inductive reasoning. We've provided hints to help you recognize the pattern in each sequence. The answers appear in the answer section.

1. Determine the next entry in the sequence.
   SSS, SCC, C, SC, __?__
   Hint: Think of the capital letters in the English alphabet. A is made up of three straight lines. B consists of one straight line and two curved lines. C is made up of one curved line.
2. Determine the next line in this sequence of digits.



2 Understand and use deductive reasoning.

Deductive Reasoning

We use inductive reasoning in everyday life. Many of the conjectures that come from this kind of thinking seem highly likely, although we can never be absolutely certain that they are true. Another method of reasoning, called *deductive reasoning*, or *deduction*, can be used to prove that some conjectures are true.

DEDUCTIVE REASONING

Deductive reasoning is the process of proving a specific conclusion from one or more general statements. A conclusion that is proved to be true by deductive reasoning is called a **theorem**.

Deductive reasoning allows us to draw a specific conclusion from one or more general statements. On the next page are two examples of deductive reasoning. Notice that in both everyday situations, the general statement from which the conclusion is drawn is implied rather than directly stated.

Everyday Situation	Deductive Reasoning
One player to another in a Scrabble game: "You have to remove those five letters. You can't use TEXAS as a word."	• All proper names are prohibited in Scrabble. (general statement) TEXAS is a proper name. Therefore, TEXAS is prohibited in Scrabble. (conclusion)
Advice to college freshmen on choosing classes: "Never sign up for a 7 A.M. class. Yes, you did it in high school, but Mom was always there to keep waking you up, and if by some miracle you do make it to an early class, you will sleep through the lecture when you get there." (*Source*: *How to Survive Your Freshman Year*, Hundreds of Heads Books, 2004)	• All people need to sleep at 7 A.M. (general statement) You sign up for a class at 7 A.M. Therefore, you'll sleep through the lecture or not even make it to class. (conclusion) In Chapter 3, you'll learn how to prove this conclusion from the general statement in the first line. But is the general statement really true? Can we make assumptions about the sleeping patterns of all people, or are we using deductive reasoning to reinforce an untrue reality assumption?

Our next example illustrates the difference between inductive and deductive reasoning. The first part of the example involves reasoning that moves from specific examples to a general statement, illustrating inductive reasoning. The second part of the example begins with the general case rather than specific examples and illustrates deductive reasoning. To begin the general case, we use a letter to represent any one of various numbers. A letter used to represent any number in a collection of numbers is called a **variable**.

A BRIEF REVIEW

In case you have forgotten some basic terms of arithmetic, the following list should be helpful.

Sum:	the result of addition
Difference:	the result of subtraction
Product:	the result of multiplication
Quotient:	the result of division

EXAMPLE 5 *Using Inductive and Deductive Reasoning*

Consider the following procedure:

> Select a number. Multiply the number by 6. Add 8 to the product. Divide this sum by 2. Subtract 4 from the quotient.

a. Repeat this procedure for at least four different numbers. Write a conjecture that relates the result of this process to the original number selected.

b. Use the variable n to represent the original number and use deductive reasoning to prove the conjecture in part (a).

SOLUTION

a. First, let us pick our starting numbers. We will use 4, 7, 11, and 100, but we could pick any four numbers. Next we will apply the procedure given in this example to 4, 7, 11, and 100, four individual cases, in **Table 1.1**.

TABLE 1.1 Applying a Procedure to Four Individual Cases

Select a number.	4	7	11	100
Multiply the number by 6.	$4 \times 6 = 24$	$7 \times 6 = 42$	$11 \times 6 = 66$	$100 \times 6 = 600$
Add 8 to the product.	$24 + 8 = 32$	$42 + 8 = 50$	$66 + 8 = 74$	$600 + 8 = 608$
Divide this sum by 2.	$\frac{32}{2} = 16$	$\frac{50}{2} = 25$	$\frac{74}{2} = 37$	$\frac{608}{2} = 304$
Subtract 4 from the quotient.	$16 - 4 = 12$	$25 - 4 = 21$	$37 - 4 = 33$	$304 - 4 = 300$